2022 Winter meeting on PDEs

This online meeting will be held on Zoom from 11th to 13th February 2022. The aim of the meeting is to provide a convenient forum for mathematicians to disseminate and discuss recent progress in the field of partial differential equations.

Organizer: Huyuan Chen, Department of Mathematics,Jiangxi Normal University, China; Quoc-Hung Nguyen, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China; and Phuoc-Tai Nguyen, Department of Mathematics and Statistics, Masaryk University, Czech Republic.

The meeting will take place over Zoom. Here is the zoom link:

Meeting ID: 924 888 5804
Passcode: AMSS2022

Speakers:
- Marie-Françoise Bidaut-Véron, University of Tours, France
- Nguyen Anh Dao, University of Economics Ho Chi Minh City, Vietnam
- Konstantinos T. Gkikas, National and Kapodistrian University of Athens Greece
- Cesar T. Ledesma, National University of Trujillo, Peru
- Moshe Marcus, Technion - Israel Institute of Technology, Israel
- Giuseppe Mingione, Università Degli Studi di Parma, Italy
- Nguyen Cong Phuc, Louisiana State University, USA
- Guohuan Qiu, Chinese Academy of Sciences, China
- Erwin Topp, University of Santiago, Chile
- Laurent Veron, University of Tours, France
- Dong Ye, East China Normal University, China
- Yi Ru-Ya Zhang, Chinese Academy of Sciences, China
- Zhitao Zhang, Chinese academy of Sciences, China
- Na Zhao, Shanghai University of Finance and Economics, China
- Feng Zhou, East China Normal University, China.

Program:

February 11th, afternoon session. Chair: Quoc-Hung Nguyen
- 16:00-16:10 (GMT +8): Opening
- 16:10-17:00 (GMT +8): Laurent Veron
- 17:05-17:55 (GMT +8): Dong Ye

February 11th, evening session. Chair: Quoc-Hung Nguyen
- 20:00-20:50 (GMT +8): Moshe Marcus
- 21:00-21:50 (GMT +8): Giuseppe Mingione
- 21:55-22:45 (GMT +8): Nguyen Cong Phuc

February 12th, afternoon session. Chair: Phuoc-Tai Nguyen
- 16:00-16:50 (GMT +8): Marie-Françoise Bidaut-Véron

17:00-17:50 (GMT +8): Zhitao Zhang

February 12th, evening session. Chair: Huyuan Chen

- 20:00-20:50 (GMT +8): Feng Zhou
- 21:00-21:50 (GMT +8): Na Zhao
- 21:55-22:45 (GMT +8): Erwin Topp

February 13th, afternoon session. Chair: Huyuan Chen

- 16:00-16:50 (GMT +8): Guohuan Qiu
- 17:00-17:50 (GMT +8): Yi Ru-Ya Zhang

February 13th, evening session. Chair: Phuoc-Tai Nguyen

- 20:00-20:50 (GMT +8): Nguyen Anh Dao
- 21:00-21:50 (GMT +8): Konstantinos Gkikas
- 21:55-22:45 (GMT +8): Cesar Ledesma

Title and Abstract

Singular solutions of some elliptic equations involving mixed absorption-reaction

Marie Françoise BIDAUT-VERON, Universitéde Tours, France

Abstract: Here we study properties of nonnegative solutions of equations in a domain of $\mathbb{R}^N$, of type

$$(1) \qquad -\Delta u + u^p - M|\nabla u|^q = 0$$

where $p > 1$, $1 < q < p$ and $M > 0$. We concentrate our analysis on the solutions with an isolated singularity, or in an exterior domain, or in $\mathbb{R}^N$. The existence of such solutions and their behaviour depends strongly on the values of p and q, and in particular according to the sign of $q - \frac{2p}{p+1}$, and when $q = \frac{2p}{p+1}$ also of the value of the parameter M which becomes a key element. The description of the different behaviours is made possible by a sharp analysis of the radial solutions of the equation.

Pointwise Gradient Estimates in Multi-dimensional Slow Diffusion Equations with a Singular Quenching Term

Nguyen Anh Dao, University of Economics Ho Chi Minh City, Vietnam

Abstract: We study an existence of weak solutions to the porous medium equation with a singular absorption term:

$$(2) \qquad \partial_t u - \Delta u^m + u^{-\beta}\chi_{\{u>0\}} = 0, \quad \text{in } \Omega \times (0, \infty),$$

where $m \geq 1$, $\beta \in (0, m)$, and Ω is a bounded domain in $\mathbb{R}^N$, $N \geq 1$.
To obtain an existence result of solutions of Eq (2), we prove a universal gradient estimate. Moreover, we also investigate the quenching phenomenon of solutions of Eq (2) in a finite

time. Precisely, we prove that such a solution of Eq (2) vanishes after a finite time, even beginning with a large initial datum.

Semilinear Elliptic Equations Involving Hardy type Potentials and Absorptions Terms

Konstantinos Gkikas, Department of Mathematics, National and Kapodistrian University of Athens, Greece

Abstract: Let $\Omega \subset \mathbb{R}^N$ ($N > 2$) be a C^2 bounded domain and $\Sigma \subset \Omega$ be a compact, C^2 submanifold without boundary, of dimension k with $0 \leq k < N - 2$. Put $L_\mu = \Delta + \mu d_\Sigma^{-2}$ in $\Omega \setminus \Sigma$, where $d_\Sigma(x) = \text{dist}(x, \Sigma)$ and μ is a parameter. We will discuss the existence and nonexistence of solutions concerning the equation $-L_\mu u + |u|^{p-1} u = 0$ in $\Omega \setminus \Sigma$ with boundary measure data. K. Gkikas was supported by Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to support Post-Doctoral Researchers" (Project Number: 59). This is a joint with P.-T. Nguyen.

Fractional elliptic problem in exterior domains with nonlocal Neumann condition

César E. Torres Ledesma, Universidad Nacional de Trujillo, Peru
Join work with Claudianor O. Alves from Universidade Federal de Campina Grande, Brasil

Abstract: In this paper we consider the existence of solution for the following class of fractional elliptic problem

$$(-\Delta)^s u + u = Q|u|^{p-1} u \quad \text{in } \mathbb{R}^N \setminus \Omega, \qquad \mathcal{N}_s u = 0 \quad \text{on } \Omega.$$

where $s \in (0, 1)$, $N > 2s$, $\Omega \subset \mathbb{R}^N$ is a bounded set with smooth boundary, $(-\Delta)^s$ denotes the fractional Laplacian operator and $\mathcal{N}_s$ is the nonlocal operator that describes the Neumann boundary condition, which is given by

$$\mathcal{N}_s u(x) = c_{N,s} \int_{\mathbb{R}^N \setminus \Omega} \frac{u(x) - u(y)}{|x - y|^{N+2s}} dy, \quad x \in \Omega.$$

Boundary value problems for a class of semilinear Schrodinger equations

Moshe Marcus, Technion - Israel Institute of Technology, Haifa, Israel.

Abstract: We consider the problem, $-L_V u + f(u) = \tau$ in a smooth domain $D \subset R^N$ with prescribed data trace $u = \nu$ on the boundary of D. Here $L_V = \Delta + V$ where V is a strongly singular potential; τ and ν are measures in D and ∂D respectively; $f \in C(R)$ is a monotone increasing function such that $f(0) = 0$. We shall consider different notions of the boundary trace and the relation between them. It is well known that, depending on the data, the above problem may have no solution. We shall discuss ways of determining the 'good' part of the data (or the 'reduced measures') for which the boundary value problem admits a solution.

Update on Nonuniform Ellipticity

Giuseppe Mingione, Università Degli Studi di Parma, Italy.

Abstract: Nonuniform Ellipticity is a classical topic in PDE, and regularity of solutions to nonuniformly elliptic and parabolic equations has been studied at length. I will present some recent results in this direction, including the solution to the longstanding issue of the validity of Schauder estimates in the nonuniformly elliptic case obtained in collaboration with Cristiana De Filippis.

A comparison estimate for singular p-Laplace equations and its consequences

Nguyen Cong Phuc, Department of Mathematics, Louisiana State University,USA.

Abstract: We present a comparison estimate for p-Laplace type equations with measure data.The main feature is that it works for all $1 < p < 3/2$ which is enough to cover the remaining open range where $1 < p \leq \frac{3n-2}{2n-1}$. Here $n \geq 2$ is the dimension of the ambient space. Pointwise estimates for solutions and their full or fractional derivatives are obtained as a consequence of such a comparison estimate. This talk is based on joint work with Quoc-Hung Nguyen.

Interior Hessian Estimate for σ_2 Equations.

Guohuan Qiu, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

Abstract: Motivated by isometric embedding problems, E.Heinz proved interior C^2 estimate for 2-d Monge-Ampere equations. In this talk, I will introduce a new pointwise approach to the 2-d Monge-Ampere equation.

A nonlocal version of the inverse problem of Donsker and Varadhan

Erwin Topp Paredes, University of Santiago, Chile

This is a joint work with Gonzalo Dávila (UTFSM-Chile).

Abstract: In their seminal paper of 1976, M.D. Donsker and S.R.S. Varadhan addressed the following "inverse problem": let consider two linear, second-order, uniformly elliptic operators L_1, L_2 with the form

$$L_i = \mathrm{Div}(A_i(x)D\phi) + b_i(x)D\phi, \quad i = 1, 2.$$

If for every domain Ω and every smooth potential V, the operators $L_1 + V$ and $L_2 + V$ have the same principal eigenvalue in Ω, then the diffusions are equal $A_1 = A_2$ and either $L_1\phi = L_2(u\phi)/u$ for some L_2-harmonic function u, or $L_2\phi = L_1(u\phi)/u$ for some L_2^*-harmonic function u. In this talk we report a nonlocal a version of this problem, where both the diffusion and transport terms defining the involved operators have a fractional nature. We prove a similar conjugacy phenomena among operators having the same principal eigenvalues, by means of a minmax characterization for them, and developing new ideas to overcome the difficulties posed by the non locality.

Boundary singular problems for mixed quasilinear equations

Laurent Véron, CNRS Institut Denis Poisson, Universitéde Tours, France

Joint work with Marie-Françoise Bidaut-Véron, Univ. Tours and Marta Garcia-Huidobro, UPC Santiago. Calc. Var. Part. Diff. Equ. to appear

Abstract: We study the existence of solutions to the problem

(3) $$-\Delta u + u^p - M|\nabla u|^q = 0 \quad \text{in } \Omega, \qquad u = \mu \qquad \text{on } \partial\Omega,$$

in a bounded domain Ω, where $p > 1$, $1 < q < 2$, $M > 0$, μ is a nonnegative Radon measure in $\partial\Omega$, and the associated problem with a boundary isolated singularity at $a \in \partial\Omega$,

(4) $$-\Delta u + u^p - M|\nabla u|^q = 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega \setminus \{a\}.$$

The difficulty lies in the opposition between the two nonlinear terms which are not on the same nature. Existence of solutions to (3) is obtained under a capacitary condition

(5) $$\mu(K) \leq c \min\left\{cap_{\frac{2}{p},p'}^{\partial\Omega}(K), cap_{\frac{2-q}{q},q'}^{\partial\Omega}(K)\right\} \quad \text{for all compact } K \subset \partial\Omega.$$

Problem (4) depends on several critical exponents on p and q as well as the position of q with respect to $\frac{2p}{p+1}$.

Hardy-Rellich inequalities revisited

Dong Ye, East China Normal University, China

Abstract: Hardy-Rellich type inequalities have broad applications in different fields of analysis and geometry, they have been studied extensively since Hardy's seminal works one century ago. In this talk, we will revise firstly various first order Hardy inequalities, and point out that most of them can be obtained by a simple and unified equality. This approach permits us to get some new or improved first order Hardy inequalities. We will explain also our approach to obtain higher order Hardy-Rellich type equalities which imply and improve many classical Hardy-Rellich inequalities. This is a joint work with Xia Huang at ECNU.

Semilinear elliptic PDEs with superlinear nonlinearity

Yi Ru-Ya Zhang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China

Abstract: Let u be a solution to the equation $-\Delta u = f(u)$, where f is postive, smooth, convex, increasing and superlinear, i.e. f(t)/t goes to $\infty ast \to \infty$. Cabrbe-Figalli-Ros-Oton-Serra proved that, when $n \leq 9$, any stable solution to this equation is bounded (and then smooth). In this talk we introduce the recent progress in this field.

Henon-Lane-Emden conjecture and related Schrodinger systems

Zhitao Zhang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences; and Jiangsu University.

Abstract: We have proved Henon-Lane-Emden conjecture is true for space dimension $N = 3$ by scaling invariant of the solutions and Sobolev embedding on S^{N-1}. Then we obtained new Liouville-type theorems and showed Henon-Lane-Emden conjecture for polyharmoic system holds in a new region, and also proved the generalized Hénon-Lane-Emden conjecture in R^2 and R^3. Moreover, we prove some new results on related Schrodinger systems.

Global Calderón–Zygmund theory for parabolic p-Laplacian system: the case $1 < p \leq \frac{2n}{n+2}$

Na Zhao, School of mathematics, Shanghai University of Finance and Economics.

Abstract: In this talk, we consider the global Calderón–Zygmund theory to parabolic p-Laplacian system:

$$u_t - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = \operatorname{div}(|F|^{p-2}F) \text{ in } \Omega \times (0, T) \subset \mathbb{R}^{n+1},$$

proving that

$$F \in L^q \Rightarrow \nabla u \in L^q,$$

for any $q > \max\{p, \frac{n(2-p)}{2}\}$ and $p > 1$. Acerbi and Mingione [*Duke Math. J., 2007*] proved this estimate in the case $p > \frac{2n}{n+2}$. In this talk, we settle the case $1 < p \leq \frac{2n}{n+2}$. We also treat systems with discontinuous coefficients having small BMO (bounded mean oscillation) norm. This is a joint work with Ke Chen and Quoc-Hung Nguyen.

Isolated singularities for fractional Lane-Emden equations

Feng Zhou, East China Normal University, China.

Abstract: We will talk about the isolated singular positive solutions for some semilinear elliptic equations, in particular, for Lane-Emden equation involving fractional elliptic operator. A classification of the isolated singularities of positive solutions is presented. Our analysis of isolated singularities is based on an integral upper bond and the study of the Poisson problem with the fractional Hardy operators. The talk is based on joint works with H.Y.Chen.